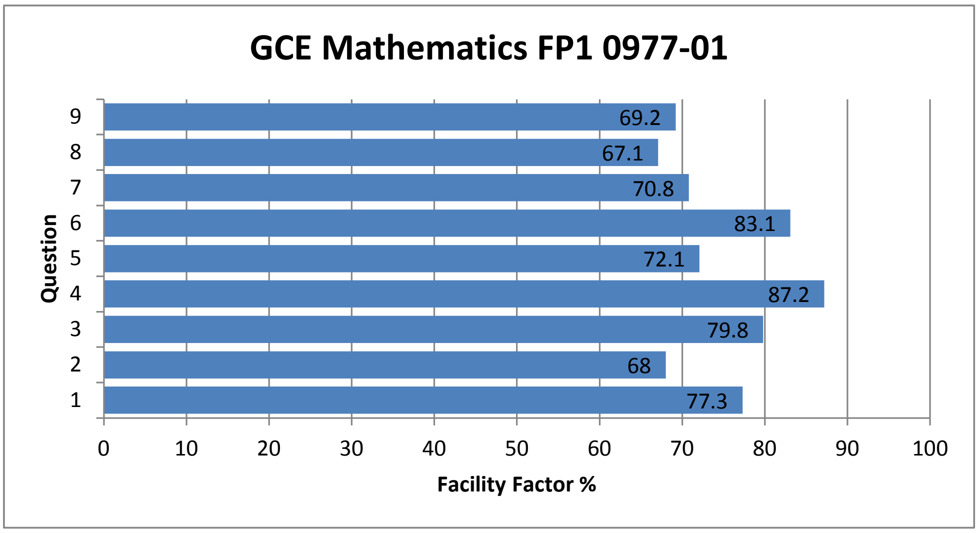
WJEC 2014 Online Exam Review

GCE Mathematics FP1 0977-01

All Candidates' performance across questions

Question Title	*N*	*Mean*	*S D*	*Max Mark*	*F F*	*Attempt %*	
1	326	7.7	2.6	10	77.3	99.7	
2	323	4.8	1.9	7	68	98.8	
3	324	6.4	2.1	8	79.8	99.1	
4	327	5.2	1.2	6	87.2	100	4
5	320	7.2	2.8	10	72.1	97.9	5b
6	324	7.5	2.2	9	83.1	99.1	
7	317	6.4	2.9	9	70.8	96.9	7
8	319	4.7	2.2	7	67.1	97.5	8
9	317	6.2	2.7	9	69.2	96.9	



4. The complex number z is given by

$$z = \frac{1 + 2\mathrm{i}}{1 - \mathrm{i}}.$$

Find the modulus and the argument of z. [6]

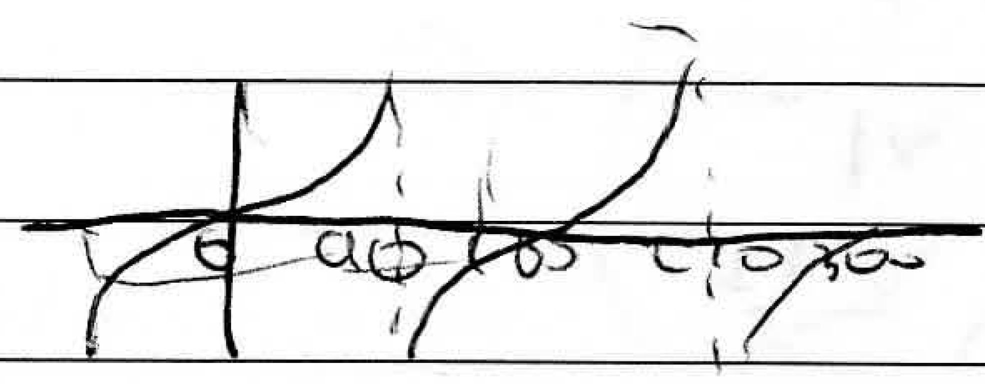
4) $z = \frac{1+2i}{1-i}$ $\qquad z = \frac{1+2i}{1-i} \times \frac{1+i}{1+i}$

$z = \frac{1+i+2i-2}{1+1}$ $\qquad z = \frac{-1+3i}{2}$

Real $x = \frac{-1}{2}$ $\qquad y = \frac{3}{2}$

$|z| = \sqrt{(-1/2)^2+(3/2)^2} = \frac{\sqrt{10}}{2} = 1.58$

$= \tan^{-1}\left(\frac{3/2}{-1/2}\right) = -71.57^\circ, 108.43^\circ, 288.43^\circ$



-1.25 radians

4) $z = \dfrac{1+2i}{1-i}$ $\quad z = \dfrac{1+2i}{1-i} \times \dfrac{1+i}{1+i}$ M1

A1

$z = \dfrac{1+i+2i-2}{1+1}$ $\quad z = \dfrac{-1+3i}{2}$ A1

Real $x = \dfrac{-1}{2}$ $\quad y = \dfrac{3}{2}$

B1

$|z| = \sqrt{(-1/2)^2+(3/2)^2} = \dfrac{\sqrt{10}}{2} = 1.58$

M1

$= \tan^{-1}\left(\dfrac{3/2}{-1/2}\right) = -71.57^\circ, 108.43^\circ, 288.43^\circ$

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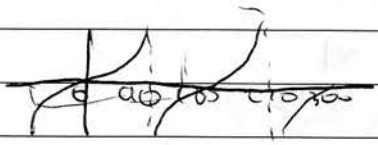

-1.25 radians

(5)

5. The roots of the cubic equation

$$x^3 + 2x^2 + 2x + 3 = 0$$

are denoted by α, β, γ.

(a) Find the cubic equation whose roots are $\beta\gamma$, $\gamma\alpha$, $\alpha\beta$. [6]

(b) Show that

$$\alpha^2 + \beta^2 + \gamma^2 = 0.$$

Deduce the number of real roots of the cubic equation

$$x^3 + 2x^2 + 2x + 3 = 0,$$

justifying your answer. [4]

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5b) $\cancel{\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2}$

$$\left[\begin{aligned}(\alpha+\beta+\gamma)^2 &= \alpha^2+\alpha\beta+\alpha\gamma+\beta\alpha+\beta^2+\beta\gamma+\alpha\gamma+\alpha\beta+\gamma^2 \\ &= \alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\beta\gamma+\alpha\gamma)\end{aligned}\right]$$

$$\begin{aligned}\alpha^2+\beta^2+\gamma^2 &= (\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\alpha\gamma) \\ &= -2^2-2(2) \\ &= 4-4 \\ &= 0 \quad \text{as required.}\end{aligned}$$

The roots squared have a sum of 0; this means there are ~~three~~ two real roots of the equation which are equal. ~~as $b^2-4ac=0$ means~~

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5b) ~~$\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2$~~

$$\left[\begin{aligned}(\alpha+\beta+\gamma)^2 &= \alpha^2+\alpha\beta+\alpha\gamma+\beta\alpha+\beta^2+\beta\gamma+\alpha\gamma+\alpha\beta+\gamma^2 \\ &= \alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\beta\gamma+\alpha\gamma)\end{aligned}\right]$$

$$\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\alpha\gamma)$$

$$= -2^2 - 2(2)$$ M1

$$= 4 - 4$$

$$= 0 \quad \text{as required.}$$ A1.

The roots squared have a sum of 0; this means there are ~~three~~ two real roots of the equation which are equal. ~~as $b^2-4ac=0$ means~~ B0

8

7. The transformation T in the plane consists of a clockwise rotation through 90° about the origin, followed by a translation in which the point (x, y) is transformed to the point $(x + 1, y + 2)$, followed by a reflection in the y-axis.

(a) Show that the matrix representing T is

$$\begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}.$$

[5]

(b) Find the equation of the image under T of the line $y = 2x + 1$. [4]

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7) T => Rotation Translation Reflection

$$\begin{bmatrix} \cos 270 & -\sin 270 & 0 \\ \sin 270 & \cos 270 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} \cos 180 & \sin 180 & 0 \\ \sin 180 & -\cos 180 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

b)
$-y - 1 = x$ $\quad y = 2x + 1$

$-x + 2 = y$

$-x + 2 = -2y - 2 + 1$

$-x + 2 = -2y - 1$

$\boxed{x - 2y - 3 = 0}$

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7) $T \Rightarrow$ Rotation Translation Reflection

$$\begin{bmatrix} \cos 270 & -\sin 270 & 0 \\ \sin 270 & \cos 270 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} \cos 180 & \sin 180 & 0 \\ \sin 180 & -\cos 180 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ ✓

$$= \begin{bmatrix} -1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$ ✓

5

b) $-y - 1 = X$ ✓ $Y = 2x + 1$

$-x + 2 = Y$ ✓

$-x + 2 = -2y - 2 + 1$

$-x + 2 = -2y - 1$

$\boxed{x - 2y - 3 = 0}$

M1

A1

A0

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8. Using mathematical induction, prove that

$$\sum_{r=1}^{n}\left(r \times 2^{r-1}\right) = 1 + 2^{n}(n-1),$$

for all positive integers n. [7]

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8. $\sum_{r=1}^{n} (r \cdot 2^{r-1}) = 1+2^n(n-1)$.

Assume that $n=k$ is true for all ~~z~~ $n \in \mathbb{Z}^+$.

$\therefore \sum_{r=1}^{k} (r \cdot 2^{r-1}) = 1+2^k(k-1)$.

Assume true for $n=k+1$ if $n=k$ is true

$\therefore \sum_{r=1}^{k+1} (r \cdot 2^{r-1}) = 1+2^{k+1}(k)$.

$\therefore \sum_{r=1}^{k+1} (\cancel{k} \cdot 2^{r-1}) = 1+2^k(k-1)+(k+1)\times 2^k$.

$= \cancel{\Sigma^k}\ 1+2^k \cdot k - 2^k + 2^k \cdot k + 2^k$.

$= 2k \cdot 2^k + 1$

$= 1+2^{k+1} \cdot (k)$

$\cancel{\therefore}$ $\cancel{\Sigma}$ $\because$ when $n=1$

$RHS = 1+2^1 \cdot (1-1) = 1$

$LHS = 1 \cdot 2^{1-1} = 1 \times 1 = 1$

$\therefore$ True for $n=1$

$\therefore$ $n=k$ is true ~~for all p~~ and $n=k+1$ is also true.

Therefore the equation is true for all positive integers.

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8. $\sum_{r=1}^{n} (r \cdot 2^{r-1}) = 1+2^n(n-1)$.

Assume that $n=k$ is true for all ~~z~~ $n \in \mathbb{Z}^+$. ✓ M1

$\therefore \sum_{r=1}^{k} (r \cdot 2^{r-1}) = 1+2^k(k-1)$.

Assume true for $n=k+1$ if $n=k$ is true ? M1

$\therefore \sum_{r=1}^{k+1} (r \cdot 2^{r-1}) = 1+2^{k+1}(k)$.

$\therefore \sum_{r=1}^{k+1} (\cancel{r} \cdot 2^{r-1}) = 1+2^k(k-1) + (k+1)\times 2^k$. ✓ A1

$= \cancel{2^k}\ 1+2^k \cdot k - 2^k + 2^k \cdot k + 2^k$.

$= 2k \cdot 2^k + 1$ ✓ A1

$= 1+2^{k+1} \cdot (k)$ ✓ A1

$\therefore$ $\cancel{\Sigma}$ $\because$ when $n=1$

RHS $= 1+2^1 \cdot (1-1) = 1$

LHS $= 1 \cdot 2^{1-1} = 1 \times 1 = 1$ B1

$\therefore$ True for $n=1$ ✓

$\therefore$ $n=k$ is true ~~for all p~~ and $n=k+1$ is also true.

Therefore the equation is true for all positive integers. A0

6